Collateral Valuation In Clearing And Settlement System

Narumon Saardchom*

One of risk management techniques for clearing and settlement system is collateral requirement. The additional market risk from potential movement in collateral value between the time of pledging and liquidating could be captured and managed by haircut system. In this paper, haircut valuation models and framework are proposed and applied to different securities classes available in Thai market. Haircut rates are calculated based on both Value-at-Risk with normal distribution and with extreme value theory. Then, the accuracies of haircut are tested using BIS Basel back-testing framework. Finally, the optimal haircut rates are chosen based on risk-cost frontier analysis.

JEL classification: C50, G28,G32

1. Introduction

Settlement risk in securities trading and derivatives markets is managed under clearing and settlement system, in which risk modeling is a critical tool for measuring an appropriate collateral arrangement. The collateral most commonly used are instruments with low credit and liquidity risk. While larger amount of collateral provides higher security for clearing and settlement system, too large required collateral may discourage trading activities and inhibit the growth of capital markets. Therefore, an efficient risk management scheme for settlement system should be the one that balances between the counterparty risk and transaction costs for market participants.

The amount of required collateral can be calculated by several methodologies, which lead to different results. Thus, the settlement risk for clearing and settlement system depend largely on the accuracy of collateral volatility valuation. In addition, there is a time lag between the time that the collateral is pledged and the time of potential default, at which point the value of collateral may decline. To prevent such additional risk, most exchanges adopt the haircut mechanism. Haircut represents the discounted value of collateral having to be pledged to take into account potential change in collateral value in the future. As a result, the higher the haircut rate, the larger collateral amount that must be pledged.

Under BIS Basel II Capital Accord, banks are allowed to use a Value-at-Risk (VaR) measure to reflect the price volatility of the exposure and collateral. The Bank of

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Thailand also adopted VaR as a measure for haircuts used in banking regulation. In addition, the SEC's system of haircuts in the US is a one-month 95% Value-at-Risk measure.

VaR can be calculated by either parametric or non-parametric approach. The non-parametric approach does not make any assumption about the shape of the loss random variable. The nature of asymmetric loss distribution can be well taken care of with this approach, but VaR figure could be sensitive to information available in selected window period. The parametric approach relies on the assumption that the loss random variable belongs to a parametric family. Usually, asset returns are assumed to belong to a normal distribution.

It is worth noting that there are some weaknesses of VaR; (i) VaR is measured with some errors from sampling variation and different statistical methodologies, (ii) VaR cannot describe the worst loss, and (iii) VaR cannot describe the loss in the left tail of the distribution. Although traditional practice and regulation are in favor of VaR for haircut valuation, VaR cannot fully capture market risk in extreme loss event, which is important for clearing and settlement system because default risk is likely to be high during an extreme event. That is, the counterparty is likely to default when the asset price largely declines. Garcia and Gencay (2006) propose an alternative approach for haircut valuation based on a VaR measure with a distribution based on extreme value theory that can better capture extreme tail loss.

In this paper, we use the Thai data to calculate and compare haircut valuation based on a VaR with normality assumption and VaR with extreme value theory. The main objective is to derive the framework for calculating the most accurate haircut rate given available data. Then, haircut results are tested by BIS Basel back-testing. Finally, we adopt the risk-cost frontier analysis proposed by Garcia and Gencay (2006) to derive the optimal haircut rates.

2. Haircut Valuation Models

The accuracy of haircut valuation is crucial for clearing and settlement system. Haircut should accurately capture volatility of security price that could be measured from the distribution of securities' return. Thus, the accuracy of haircut valuation depends largely on the choice of probability distribution. Both SEC and Basel II capital accord recommend the use of VaR measure for haircut valuation because it could provide one figure representing the maximum price volatility with a given probability over a given time horizon.

2.1. VaR With Normality Assumption

There are two main approaches for VaR calculation—parametric and non-parametric models. A non-parametric model makes no assumption on the distribution of securities' return, but derive the distribution based on historical return data. The VaR result of a non-parametric model may fail to capture the extreme loss that is not included in the chosen data sample. Moreover, a VaR measure becomes more uncertain at higher confidence level. In contrast, a parametric model assumes that the securities' return follow a parametric family such as a normal distribution. The VaR measure under a parametric approach is calculated by a following equation.

$$\mathrm{VaR}(\alpha) = \mu \pm \sigma \cdot z(\alpha)$$

Where 1-α is a confidence level, μ is expected securities' return, σ is volatility of securities return, and z(α) is α quantile of standard normal cumulative probability.

Most clearinghouses rely on a parametric approach for haircut valuation based on VaR by assuming that securities return has a normal distribution. The disadvantage of this approach is that the VaR result will no longer be accurate when securities' return does not have a normal distribution. Moreover, a VaR measure cannot capture extreme loss events, which is very important for clearing and settlement system because counterparty risk is likely to be high when the security prices drop significantly or when the securities' returns are from the fat tail distribution. In such condition, haircut measure from VaR under normality assumption can be a critical source of error.

2.2. VaR With Extreme Value Theory

McNeil (1999) and Garcia and Gencay (2006) propose an alternative approach, a VaR measure with the return distribution based on Extreme Value Theory (EVT) method, for haircut valuation of a fat tail return distribution.

Since we are interested in the distribution of securities' return in the left tail of the distribution, we adopt a "peak-over-threshold" method to derive the probability distribution of the left tail. For a given threshold u, the probability that the securities' return will exceed the level *u* is defined as:

$$F_u(y) = \Pr\left(r - u \le y \mid r > u\right) \text{for } y \ge 0, r = y + u$$

$$F_u(y) = \frac{F(y+u) - F(u)}{1 - F(u)}$$

If the threshold u is large enough, the distribution of the return exceeding *u* can be approximated by Generalized Pareto Distribution (GPD) with the following relationship.

$$F_u(y) = G_{\xi,\sigma}(y)$$

The distribution of r can then be simplified as:

$$F(r) = (1 - F(u))G_{\xi,\sigma}(y) + F(u)$$

Therefore, F(u) can be estimated by $(N-n_u)/N$ where N is the number of observations with negative return and n_u is the number of observations with return exceeding a threshold u. The cumulative density function of Generalized Pareto distribution, $G_{\xi,\sigma}(y)$is defined as:

$$G_{\xi,\sigma}(y) = \begin{cases} 1 - \left(1 + \dfrac{\xi y}{\sigma}\right)^{-1/\xi} & \text{where } \xi \neq 0 \\ 1 - e^{(-y/\sigma)} & \text{where } \xi = 0 \end{cases}$$

Where ξ is a shape parameter and σ is a scale parameter. Then, the distribution of r can be revised as follow.

$$F(r) = \left(1 - \left(\frac{N - n_u}{N}\right)\right)\left(1 - \left(1 + \frac{\xi y}{\sigma}\right)^{-1/\xi}\right) + \frac{N - n_u}{N}$$

$$F(r) = 1 - \frac{n_u}{N}\left(1 + \frac{\xi y}{\sigma}\right)^{-1/\xi}$$

Therefore, the VaR measure with the distribution based on extreme value theory method can be derived as:

$$VaR(\alpha) = u + \frac{\sigma}{\xi}\left[\left(\frac{N}{n_u}\alpha\right)^{-\xi} - 1\right]$$

ξ and σ can be estimated by a maximum likelihood method. Note that the probability density function of Generalized Pareto Distribution is represented by the following equation.

$$f(y) = \frac{1}{\sigma}\left(1 - \frac{\xi y}{\sigma}\right)^{\frac{1}{\xi} - 1} \text{ where } \xi \neq 0$$

As a result, the likelihood function can be shown as:

$$L(\xi, \sigma | y) \prod_{i=1}^{n} \frac{1}{\sigma}\left(1 - \frac{\xi y_i}{\sigma}\right)^{\frac{1}{\xi} - 1}$$

Thus, the log likelihood is

$$\ln(L(\xi, \sigma | y)) = -\frac{n \xi \ln(\sigma) + (\xi - 1)\sum_{i=1}^{n} \ln\left(1 - \frac{\xi y_i}{\sigma}\right)}{\xi}$$

We can then solve the following equations for maximum likelihood estimators for ξ and σ.

$$\frac{\partial \ln L(\xi, \sigma | y)}{\partial \xi} = 0 \text{ and } \frac{\partial \ln L(\xi, \alpha | y)}{\partial \sigma} = 0$$

2.3. Back-testing

We test the result of a haircut valuation by BIS Basel back-testing. The BIS Basel back-testing method is used to test the accuracy of the model by testing the hypothesis whether the probability that the security price declines under haircut level more than α%. That is,

H_0: Pb = α
H_1: Pb > α

where $$Pb(n, x : \alpha) = \binom{n}{x} (1 - \alpha)^{n-x} \alpha^{x}$$

n = number of observations
x = number of observations that the security price decline under haircut level
α= the probability that the security price declines under haircut (assigned to 0.01)

There are three possible back-testing results;

a. Green zone—if Pb $\geq$ 5%, the hypothesis cannot be rejected. Thus, the haircut result is acceptably accurate.

b. Yellow zone—if 0.1% < Pb < 5%, the haircut result will be acceptable with BIS adjusted factor equal to $z_{\alpha}/z_{\alpha^{\cdot}}$ where α is equal to x/n.

c. Red zone—if Pb $\leq$ 0.1%, the haircut result is not acceptable. The new model or new method is required.

3. Haircut Results

In this paper, we retrieve equity and bond daily price data from the Thai markets. For traded securities in the Stock Exchange of Thailand, daily prices of three groups of equity; SET50 index, non-SET50 Index, and warrants, are selected from the same window period; 6 February 2006 to 28 December 2008. There are 464 observations for each group of securities. For government bonds, five classes of daily bond price data, based on maturity[1] (less then 1 year, 1 to 3 years, 3 to 7 years, 7 to 10 years, and more than 10 years), are taken from 4 January 2000 to 31 July 2008.

We test the normality of securities return distribution with histogram and descriptive statistics for all acceptable collaterals. The VaR measures with both normality assumption and extreme value theory are then calculated along with BIS back-testing to confirm the accuracy of VaR measures for haircut results.

3.1. Normality Assumption

To test a normality assumption on securities return, we plot the histogram of equity and bond returns, shown in figure 1 and 2, and calculate descriptive statistics. Most securities have excess kurtosis larger than zero. The tail distribution of both SET50 index and government bonds with all maturities appears to follow a normal distribution. However, returns of stocks in non-SET50 have much longer and heavier tail than suggested by a normal distribution. This implies that haircut valuation based on VaR with normality assumption may not be appropriate for stocks in non-SET50.

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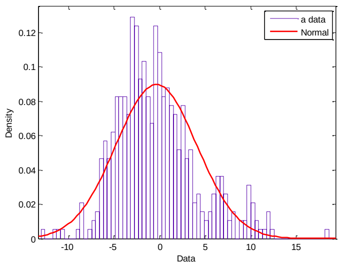
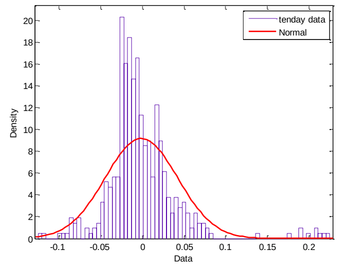
[1] ThaiBMA bond maturity classification

Figure 1: Histogram of Equity Returns

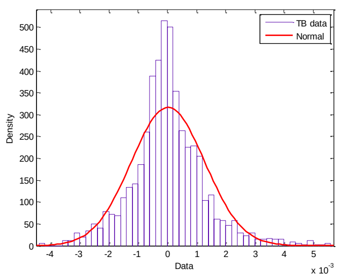


(a) SET 50

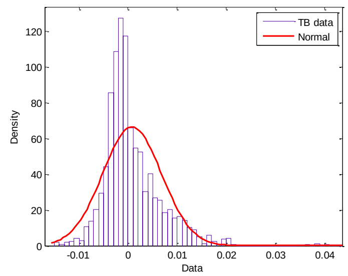


(b) Non-SET50

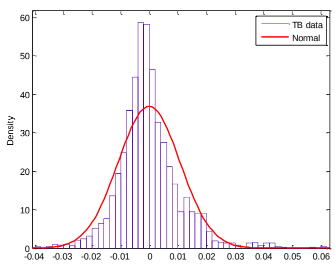
Figure 2: Histogram of Government Bond Returns



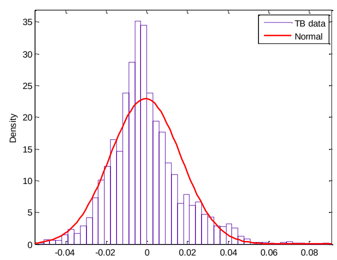
(a) maturity < 1 year



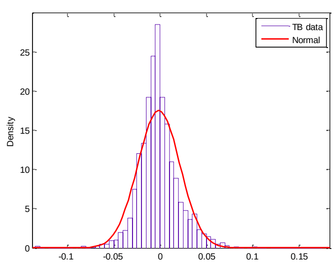
(b) maturity 1-3 years



(c) maturity 3-7 years



(d) maturity 7-10 years



(e) maturity > 10 years

3.2. VaR With Normality Assumption

Assume that common stocks' returns are normally distributed with mean μ and standard deviation σ. The result of VaR with normality assumption for SET50 index and stocks in non-SET50 with 10-day horizon and 99% confidence level are shown in table 9. Note that the result of haircut rates for stocks in non-SET50 calculated by VaR with normality assumption may not be appropriate given the result of histogram and descriptive statistics in the previous section.

Table 9 VaR with Normality Assumption for Common Stocks

	SET50	Non-SET50
Average	2.37%	0.25%
Standard Deviation	4.22%	5.03%
VaR(99%)	**12.20%**	**11.94%**

We classify warrants into three classes based on their liquidity—highly liquid, semi-liquid, and illiquid warrants. Highly liquid, semi-liquid, and illiquid warrants are warrants that were traded more than 80% , from 60% to 80%, and less than 60% of the total number of trading days in data sample, respectively. From the total number of warrants of 71, there are only 3 warrants in semi-liquid class and 14 warrants in illiquid class. The haircut results based on 10-day 99% VaR with normality assumption are 50.73%, 55.06%, and 89.86% for highly liquid, semi-liquid, and illiquid warrants, respectively. It is important to note that the haircut results from a traditional VaR measure can mainly capture market risk. To take into account liquidity risk, the VaR measure was therefore adjusted upward by increasing horizon to 10 days.

The results of VaR with normality assumption for government bond are shown in table 10.

Table 10: VaR with Normality Assumption for Government Bond

	Remaining Time to Matuarity				
	< 1 year	1 - 3 years	3 - 7 years	7 - 10 years	> 10 years
Mean	0.0280%	0.1298%	0.3565%	-0.5105%	-0.4999%
Stdev	0.2313%	0.5085%	1.5489%	2.5602%	3.1275%
VaR (99%)	0.5660%	1.3127%	3.9598%	5.4455%	6.7756%

3.3. VaR With Extreme Value Theory

For different level of chosen threshold, u, we estimate shape and scale parameters of Generalized Pareto distribution by maximum likelihood method and calculate associated VaR measures shown in table 11.

Table 11: VaR with Extreme Value Theory for Common Stocks in SET50 Index

U	0.00%	2.00%	4.00%	6.00%	8.00%	10.00%
σ	4.57%*** (0.0044)	4.42%*** (0.0052)	4.72%*** (0.0066)	3.06%*** (0.0055)	3.00%*** (0.0073)	1.17%** (0.005)
ξ	-0.1714*** (0.0628)	-0.1952*** (0.0714)	-0.2695*** (0.0723)	-0.1308 (0.1086)	-0.1638 (0.1357)	0.2749 (0.3346)
n_u	182	113	65	49	24	13
N	182	182	182	182	182	182
VaR (99%)	**14.55%**	**14.53%**	**14.83%**	**14.19%**	**14.31%**	**13.05%**

* Significant at 90%, ** Significant at 95%, *** Significant at 99%

For common stocks in SET50 index, the values of shape parameter estimates, ξ, are significantly less than zero, which implies that the tail of the distribution may be shorter than that of normal distribution. Therefore, VaR with normality assumption is more suitable for common stocks in SET50 index than VaR with extreme value theory.

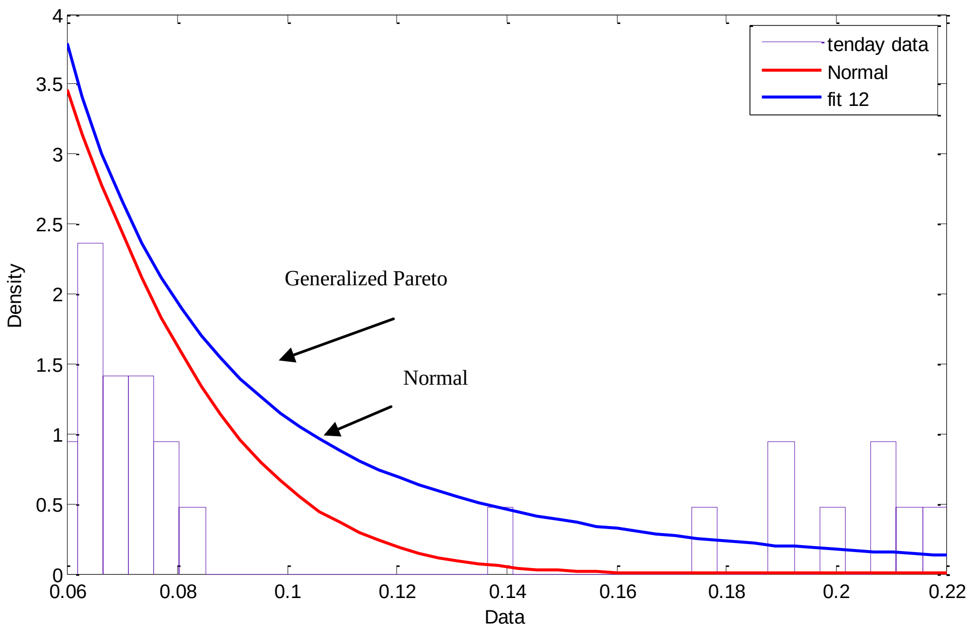
Table 12: VaR with Extreme Value Theory for Common Stocks in Non-SET50

U	0.00%	2.00%	4.00%	6.00%
σ	2.99%*** (0.0032)	2.32%*** (0.004)	2.38%*** (0.0063)	3.57% (0.0214)
ξ	0.1649** (0.0774)	0.4029*** (0.1472)	0.5501** (0.2365)	0.5518 (0.5727)
n_u	175	95	49	24
N	175	175	175	175
VaR (99%)	**20.62%**	**25.03%**	**26.73%**	**26.97%**

* Significant at 90%, ** Significant at 95%, *** Significant at 99%

On the other hand, table 12 shows that the value of shape parameter estimates, ξ, for common stocks in non-SET50, are all significantly greater than zero, which implies that the tail of the distribution is longer than that of normal distribution. These results confirm the findings from histogram and descriptive statistics that VaR with normality assumption may underestimate the actual loss. Therefore, VaR with extreme value theory is preferred for common stocks in non-SET50. Figure 3 shows that the tail distribution of stock returns in non-SET50 can be better explained by a GPD than by a normal distribution.

Figure 3: Non-SET50 tail Distribution: GPD vs. Normal Distribution



4. Haircut Rate and Back-testing

4.1. Haircut Rates

The results of haircut rates are summarized in table 13 and 14. Given the results from a previous section, haircut rates for common stocks in SET50 and warrants are the ones based VaR measures with normality assumption whereas haircut rate for common stocks in non-SET50 is the one based on VaR measures with extreme value theory. For bonds, haircut rates are calculated based on VaR with normality assumption.

Table 13: Haircut Rates for Equity

	Stock		Warrant		
	SET50	Non-SET50	Illiquid	Semi Liquid	Highly Liquid
VaR (99%) with					
Normality	12.20%	-	89.86%	55.06%	50.73%
Extreme Value Theory	-	26.73%	-	-	-

Table 14: Haircut Rates for Government Bond

	Remaining Time to Maturity				
	< 1 Year	1- 3 Years	3 - 7 Years	7 - 10 Years	> 10 Years
VaR (99%) with					
Normality	0.57%	1.31%	3.96%	5.45%	6.78%
Extreme Value Theory	-	-	-	-	-

4.2. Back-Testing

Finally, we conduct a back-testing based on BIS Basel on all haircut rates. For equity back-testing, we use the data from 6 February 2006 to 28 December 2007 with the total number of 464 trading days. Given 464 observations, the BIS Basel back-testing result will fall into red, yellow, and green zones when the number of observations that the security price decline under haircut level are 8, 9 to 12, and 13 or more.

4.2.1. Common Stocks in SET50 Index and Non-SET50 Index

There are only two days that the returns of stocks in SET50 index fall below the haircut level of 12.20%, shown figure 4. As a result, the haircut rate of 12.20% for stocks in SET50 index is in the green zone or is acceptably accurate based on BIS Basel back-testing. Figure 5 shows that the haircut rate for stocks in non-SET50 index also falls into a green zone of BIS Basel back-testing, where there is zero day that the return of stocks in non-SET50 index falls under the haircut rate of 26.73%.

Figure 4: BIS Basel Back-testing for Stocks in SET50 Index

(Haircut = 12.20% by VaR with Normality Assumption)

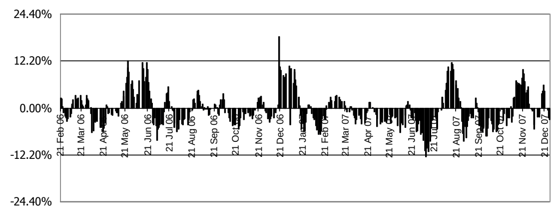
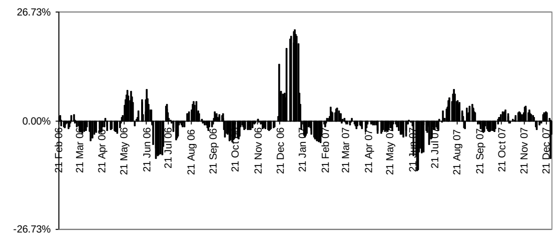


Figure 5: BIS Basel Back-testing for Stocks in Non-SET50

(Haircut = 26.73% by VaR with Extreme Value Theory)



4.2.2. Warrants

Figure 6, 7, and 8 show that haircut rates for all three classes of warrants, highly liquid, semi-liquid, and illiquid, are acceptably accurate based on BIS Basel back-testing. There are only 4 out of 464 days that the return of highly liquid warrants fall under haircut rates of 50.73%. For semi-liquid and liquid warrants, there are 6 and 4 out of 464 days that the warrant lost its value more than haircut rates of 55.06% and 89.86%, respectively.

Figure 6: BIS Basel Back-testing for Highly-liquid Warrant

(Haircut = 55.06% by VaR with Normality Assumption)

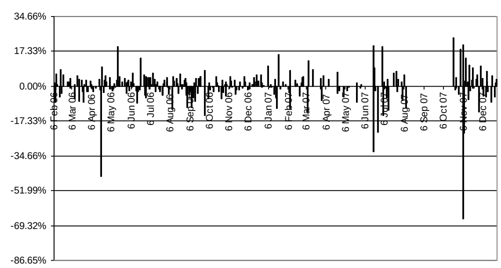


Figure 7: BIS Basel Back-testing for Semi-liquid Warrant

(Haircut = 50.73% by VaR with Normality Assumption)

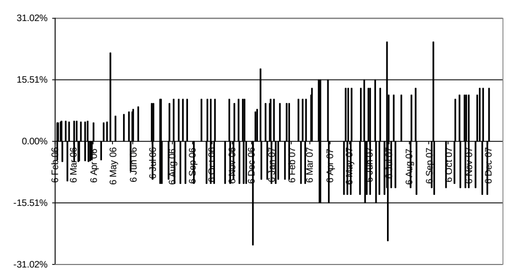
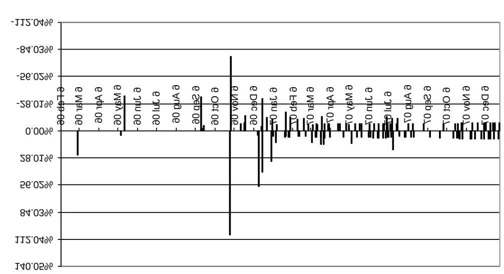


Figure 8: BIS Basel Back-testing for Illiquid Warrant

(Haircut = 89.86% by VaR with Normality Assumption)



4.2.3. Government Bond

The number of observations of government bond with each maturity class and their associated expected and actual number of exceptions, which are the number of observations that the government bond price declines by more than their haircut rates, under the BIS Basel back-testing framework are shown in table 15.

Table 15: BIS Basel Backtesting for Government Bond

Maturity (year)	**Number of observations**	**Number of Exceptions**			**Actual Number of Exceptions**
		Green	**Yellow**	**Red**	
<1	1728	21	22-30	31 or more	0
1-3	2341	27	28-38	39 or more	85
3-7	2341	27	28-38	39 or more	19
7-10	2341	27	28-38	39 or more	12
>10	2293	26	27-37	38 or more	16

The actual number of exceptions indicates that BIS Basel back-testing results are in green, yellow, and red zone. In addition, figure 9 to 13 show the graphical BIS Basel back-testing results for each maturity class of government bonds.

Figure 9: BIS Basel Back-testing for Government Bond (< 1 yr)

(Haircut = 0.57% by VaR with Normality Assumption)

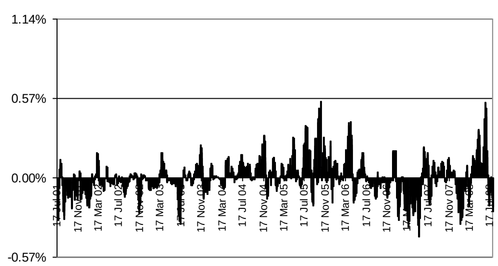


Figure 10: BIS Basel Back-testing for Government Bond (1- 3 yr)

(Haircut = 1.31% by VaR with Normality Assumption)

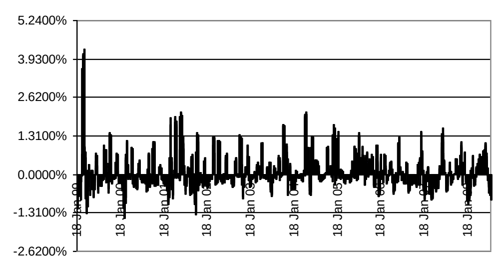


Figure 11: BIS Basel Back-testing for Government Bond (3-7 yr)

(Haircut = 3.96% by VaR with Normality Assumption)

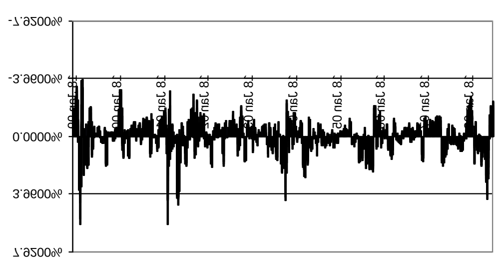


Figure 12: BIS Basel Back-testing for Government Bond (7-10 yr)

(Haircut = 5.45% by VaR with Normality Assumption)

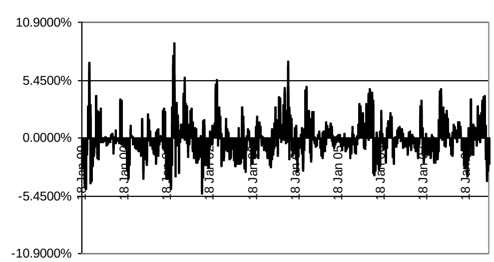


Figure 13: BIS Basel Back-testing for Government Bond (>10 yr)

(Haircut = 6.78% by VaR with Normality Assumption)

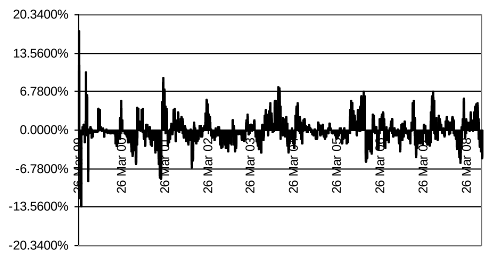
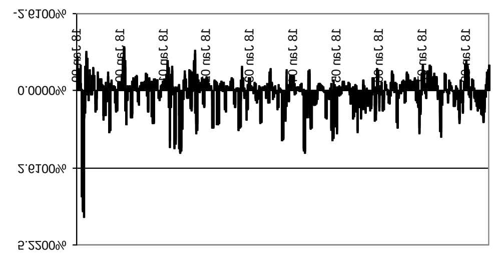


Figure 14: BIS Basel Back-testing for Government Bond (1- 3 yr)

(Haircut = 2.61% by VaR with Extreme Value Theory)



Except the class with 1 to 3 years to maturity that its back-testing result falls under the BIS Basel red zone, haircut rates for government bonds with all other maturity classes are acceptable accurate under the BIS Basel back-testing. The haircut rate of government bond with 1 to 3 years to maturity is thus recalculated by a new method, VaR with extreme value theory. Consequently, the haircut rate for this maturity class increases from 1.31% to 2.61%. We repeat BIS Basel back-testing process on this new haircut rate, shown in the figure 14, and find that it is in the BIS Basel green zone with 10 actual exceptions.

5. Benefits and Costs of Collateral

The main benefit of collateral is credit risk reduction by liquidation value of collateral in the event of default even though the liquidation value may be less than its expected value. Collateral should also reduce the probability of default because the more collateral pledged, the larger loss amount experienced by the provider in the default event. As a result, higher haircut rates are preferable for clearing and settlement system. However, higher haircut rates increase the transaction costs for market participants and could have a negative effect on market growth.

Although VaR with extreme value theory can provide a larger haircut rates than that derived by VaR with normality assumption, we propose that VaR with extreme value theory is used only when the tail distribution of securities' return cannot be appropriately approximated by a normal distribution. We find that VaR with normality assumption is appropriate for common stocks in SET50 index, warrants, and government bonds with all maturities except for government bonds with maturity of 1 to 3 years, of which VaR with extreme value theory provides a better fit for haircut rate. For common stocks in non-SET50, VaR with extreme value theory can provide a more accurate haircut rate.

Finally, to assure that the haircut rates derived from VaR with extreme value theory are not too large; we perform a risk-cost frontier analysis to confirm the optimal haircut solutions. Haircut rates and tail risk are compared at different levels of possible thresholds. The optimal haircut rate should be at the level that leads to an acceptable level of tail risk, but not too large for market participants.

Figure 15 and 16 exhibit risk-cost frontier analysis for common stocks in non-SET50 and for government bond with maturity of 1 to 3 years. The results of risk-cost frontier confirm that, for all possible threshold levels, haircut rates by VaR with extreme value theory; 26.73% for common stocks in non-SET 50 and 2.61% for government bond with maturity of 1 to 3 years, are associated with the tail risk not over 1% or at 99% confidence level.

Figure 15

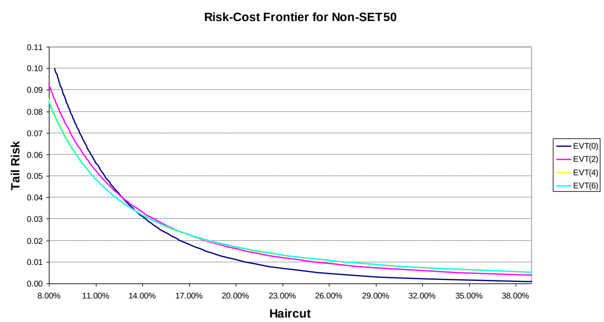
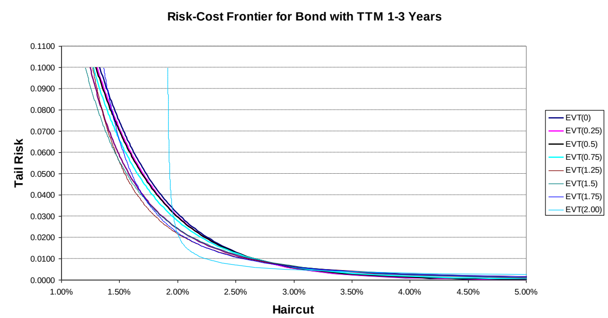


Figure 16



6. Conclusion

Collateral plays a very important role in mitigating credit risk in clearing and settlement system. Since there is a possible change in value of collateral from pledging and liquidating dates, haircut rate has been adopted in almost all exchanges to manage such additional market risk in potential default events.

For most exchanges, a traditional approach for haircut valuation has been based on VaR with normality assumption. However, when the tail distribution of securities' return cannot be fully explained by a normal distribution, the accuracy of haircut rates is in question. That is, when the securities' return exhibits a longer and heavier tail than that of a normal distribution, haircut rates calculated by VaR with normality assumption will underestimate the true additional market risk in clearing and settlement system. An alternative approach to obtain a more accurate haircut rates is VaR with extreme value theory, where the tail of securities' returns are assumed to follow Generalized Pareto Distribution (GPD) instead of a normal distribution.

The optimal haircut rate should be the one that balances between the risk of clearing and settlement system and the transaction costs for market participants. We consider both equity and debt securities that are acceptable as collaterals. We use the Thai data to study a haircut valuation. Common stocks are split into two groups: common stocks in SET50 index and stocks in non-SET50 index. Warrants are classified into three classes according their liquidity; highly liquid, semi-liquid, and illiquid. Government bonds are categorized into five classes based on their maturity.

Haircut rates for each type of collateral are calculated with two main approaches: VaR with normality assumption and VaR with extreme value theory based on GPD. We conduct BIS Basel back-testing on all haircut rates and find that VaR with normality assumption is appropriate for a haircut valuation of common stocks in SET50 index, warrants, and government bonds with maturity of less than 1 year, 3 to 7 years, 7 to 10 years, and longer than 10 years, while VaR with extreme value theory is appropriate for a haircut valuation for stocks in non-SET50 index and government bonds with maturity of 1 to 3 years.

We conduct a risk-cost frontier analysis for haircuts calculated by VaR with extreme value theory. The result confirms that the haircut rates are associated with the desired confidence level at 99%. Although this paper deals with the Thai data, we hope that it provides a framework for haircut valuation in other markets as well as a fundamental background for related applications.

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